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COMPUTATION OF THE DYNAMIC GREEN'S TENSOR OF A STOCHASTICALLY INHOMOGENEOUS ELASTIC MEDIUM

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The problem of finding mean Green's tensor of the microinhomogeneous elastic unbounded medium is considered. In the case of a statistically isotropic homogeneous medium the problem reduces to that of finding the eigenvalues of the elastic and polarization operators and of calculating the inverse Fourier transforms. The method of changing the field variables was used to sum all one-point and two-point sequences in the expansion for the elastic operator and its eigenvalues. A general expression for the mean Green's tensor is obtained for a particular correlation function. The problems of obtaining an approximate expression for this tensor in terms of the first roots and of finding the asymptotic formulas in terms of the wavelengths (frequencies) are discussed.

Methods of obtaining the Green's function for the inhomogeneous and randomly-inhomogeneous media were discussed in [1 — 5]. Use of the methods of the Green's function in stochastic systems is the subject of [6 — 8].

1. Let us consider an unbounded linear elastic inhomogeneous medium in which the stresses σ_{ij} and deformations e_{ij} are connected by the following relation:

$$\sigma = \lambda(x)e, \quad e_{kl} = {}^1/_2 (u_{kl,} + u_{l,k}) \tag{1.1}$$

where $\lambda_{ijkl}(x)$ is the tensor of elastic coefficients, depending on the spatial coordinates in a random manner. Here and in the following we shall use a straightforward notation for the field variables passing, when necessary, to a coordinate representation of the tensors. The Green's tensor $G_{kn}(x, x_1)$ of the medium in question satisfies the equation

$$(\lambda G_{,x})_{,x} + \rho_0\omega^2 G = \delta(x - x_1) \tag{1.2}$$

Bringing into consideration an auxiliary medium with the parameters λ°, ρ_0, we pass from the equation (1.2) to the equivalent integral equation

$$G(x, x_1) = G^\circ(x - x_1) - \int G^\circ(\lambda' G_{,x})_{,x}\, dx_1, \quad \lambda' = \lambda - \lambda^\circ \tag{1.3}$$

($G_{kn}^\circ(x - x_1)$ denotes the Green's tensor of the homogeneous auxiliary medium). Differentiating (1.3), transferring the derivative under the integral sign and writing [9]

$$\overset{\circ}{G}_{,xx}(x - x_1) = G^{(s)}_{,xx}\,\delta(x - x_1) + G^R_{,xx}(x - x_1)$$

we obtain

$$BG_{,x} = \overset{\circ}{G}_{,x} - \int G^{(R)}_{,xx}\lambda' G_{,x} dx_1$$
$$B_{iqjl} = \delta_{iq}\delta_{jl} + \lambda'_{nmql} G^{(s)}_{in,mj}$$

Passing to new field variables g and Γ [9]

$$g_{ijk} = B_{iqjl} G_{qk,l}, \quad \Gamma_{nmst} = \lambda'_{nmql} B^{-1}_{qlst} \tag{1.4}$$

we can write the integral equation in the form

$$g = \overset{\circ}{G}_{,x} - \int G^{(R)}_{,xx}\Gamma g dx_1 \tag{1.5}$$

Solving (1.5) by consecutive iterations, we obtain an expression for g_{ijk} in the form of a dispersion series in powers of Γ. The best convergence of the series is obtained when [9]

$$\langle \Gamma_{ijkl} \rangle = 0 \tag{1.6}$$

A direct check shows that when (1.6), $\langle g \rangle$ satisfies the equation

$$\langle g \rangle = \overset{\circ}{G}_{,x} + \int\int G^{(R)}_{,xx} Q \langle g \rangle dx_1 dx_2 \tag{1.7}$$
$$Q_{nmst}(x_1, x_2) = \langle \Gamma_{nmpq}(x_1) \Gamma_{\beta\gamma st}(x_2) \rangle G^{(R)}_{p\beta,q\gamma}(x_2 - x_1) + \ldots$$

We define the effective polarization tensor Γ^*_{nmst} by the relation

$$\langle \Gamma g \rangle = \Gamma^* \langle g \rangle = \int \gamma^*(x, x_1) \langle g \rangle dx_1 \tag{1.8}$$

Averaging the equation (1.5) and taking (1.8) into account, we obtain

$$\langle g \rangle = \overset{\circ}{G}_{,x} - \int G^{(R)}_{,xx} \Gamma^* \langle g \rangle dx_1 \tag{1.9}$$

Equating (1.7) and (1.9), we obtain the relation connecting γ^* and Q

$$\gamma^*(x, x_1) = -Q(x, x_1) \tag{1.10}$$

The field quantities $G_{,x}, \lambda$ and g, Γ are connected by the formulas

$$g = (E^1 + \lambda' G^{(s)}_{,xx}) G_{,x} \tag{1.11}$$
$$\Gamma g = \lambda' G_{,x}, \quad E^1_{iqjl} = \delta_{iq}\delta_{jl}$$

Averaging (1.11) and taking (1.8) into account, we obtain

$$\langle g \rangle = \langle G_{,x} \rangle + G^{(s)}_{,xx} \Lambda' \langle G_{,x} \rangle \tag{1.12}$$
$$\Gamma^* \langle g \rangle = \Lambda' \langle G_{,x} \rangle, \quad \Lambda' = \Lambda^* - \lambda^\circ$$

where the elastic operator Λ^* is given by the relation

$$\langle \lambda G_{,x} \rangle = \Lambda^* \langle G_{,x} \rangle = \int \lambda^* \langle G_{,x} \rangle dx_1 \tag{1.13}$$

From (1.12) follows

$$\Gamma^* (E^1 + G^{(s)}_{,xx} \Lambda') \langle G_{,x} \rangle = \Lambda' \langle G_{,x} \rangle \tag{1.14}$$

Let the medium in question be statistically homogeneous and isotropic . Then

$$\gamma^*(x, x_1) = \gamma^*(x - x_1), \quad \lambda^*(x, x_1) = \lambda^*(x - x_1).$$

Let us denote the Fourier transforms of the kernels γ^* and λ^* by $D^*(\omega, k)$, $L^*(\omega, k)$. Applying the Fourier transformation to (1.14), we obtain

$$L^*(\omega, k) = \lambda^\circ + M^{-1}D^*(\omega, k), \quad M_{pnqj} = E^1_{pnqj} - D^*_{njim}G^{(s)}_{ip,\,mq} \tag{1.15}$$

Let us now average the equation (1.2), with (1.3) taken into account, and Fourier transform the result. We obtain

$$(L^*_{ijkl}k_lk_j - \rho_0\omega^2\delta_{ik})\,P_{kn} = -\,\delta_{in} \tag{1.16}$$

where $(P_{kn}\,(\omega, k)$ is a Fourier transform of the tensor G_{kn} .

2. Let the auxiliary medium be homogeneous and isotropic

$$\overset{\circ}{\lambda}_{ijkl} = \lambda_0 E^1_{ijkl} + 2\mu_0 E^2_{ijkl}, \quad E^2_{ijkl} = {}^1/_2\,(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}) \tag{2.1}$$

Then the expression for the tensor $G_{kn}{}^\circ$ is known [1].

Let us restrict ourselves in the expression (1.7) to the two-point moment $\Gamma_{ijkl}(x)$, which corresponds to the process of taking into account the two-point moments $\lambda_{ijkl}(x)$ of every order. Consider the inhomogeneous isotropic medium

$$\begin{aligned} \lambda_{ijkl}(x) &= \lambda(x)\,E^1_{ijkl} + 2\mu(x)\,E^2_{ijkl} \\ \Gamma_{ijkl}(x) &= \Gamma_1(x)\,E^1_{ijkl} + 2\Gamma_2(x)\,E^2_{ijkl} \end{aligned} \tag{2.2}$$

We assume that

$$\langle \Gamma_i\,(x_1)\Gamma_j\,(x_2)\rangle = R\,(0)e^{-\tau}\tau^{-1}\sin\tau, \quad \tau = a^{-1}\rho = a^{-1}\mid x_1 - x_2 \mid \tag{2.3}$$

Applying the Fourier transformation according to the second formula of (1.7) and taking (1.10), (2.3) into account, we obtain

$$D^*_{nj\gamma\delta}(\omega, k) = \sum_{\alpha=1}^{6} D_\alpha{}^*(\omega, k)\,E^\alpha_{nj\gamma\delta} \tag{2.4}$$

Formulas (1.5) with (2.4) taken into account, yield

$$\begin{aligned} L_{nj\gamma\delta}(\omega, k) &= \sum_{\alpha=1}^{6} L_\alpha{}^*(\omega, k)\,E^\alpha_{nj\gamma\delta} \\ E^3_{nj\gamma\delta} &= q_nq_j\delta_{\gamma\delta}, \quad E^4_{nj\gamma\delta} = q_\gamma q_\delta\delta_{nj} \\ E^5_{nj\gamma\delta} &= {}^1/_4\,(\delta_{n\gamma}q_jq_\delta + \delta_{n\delta}q_jq_\gamma + \delta_{j\gamma}q_nq_\delta + \delta_{j\delta}q_nq_\gamma) \\ E^6_{nj\gamma\delta} &= q_nq_jq_\gamma q_\delta, \quad q_i = k_i\,/\,k \end{aligned} \tag{2.5}$$

In the case of strong isotropy $D_i{}^*(\omega, k) = L_i{}^*(\omega, k) = 0$ $(i = 3, 4, 5, 6)$, we write the formulas (1.15) as follows:

$$\begin{aligned} L_2{}^* &= \mu_0 + \frac{D_2{}^*}{1 - T^{(t)}D_2{}^*}, \quad T^{(t)} = \frac{2\,(3\lambda_0 + 8\mu_0)}{16\mu_0\,(\lambda_0 + 2\mu_0)} \\ K^* &= K_0 + \frac{D^*}{1 - TD^*}, \quad T = (\lambda_0 + 2\mu_0)^{-1} \end{aligned} \tag{2.6}$$

$$K^* = L_1^* + {}^2/_3 L_2^*, \quad D^* = D_1^* + {}^2/_3 D_2^*$$

$$D_1^*(\omega, k) = \frac{2A}{15}\Big\{2\Psi_j^{(1)} + 2\Psi_j^{(2)} + \Psi_{j\alpha}^{(3)} + \Psi_{j\alpha}^{(4)} + \frac{3}{2}\Psi_{j\alpha}^{(6)} + \Psi_{j\alpha}^{(7)} + 2\Psi_{jl}^{(8)} + \Psi_j^{(9)}\Big\}_2^1\Big|_t^l \tag{2.7}$$

$$D_2^*(\omega, k) = \frac{2A}{15}\{\Psi_j^{(1)} + \Psi_j^{(2)} + \Psi_{j\alpha}^{(3)} + \Psi_{j\alpha}^{(4)} + \Psi_{j\alpha}^{(5)} + \Psi_{j\alpha}^{(6)} + \Psi_{j\alpha}^{(7)} + \Psi_{jt}^{(8)} + \Psi_{j\alpha}^{(10)}\}_2^1\big|_t^l$$

$$\Psi_j^{(1)} = -\frac{ik_t k^2}{c_t^2 \alpha_j^2} F\left(\frac{3}{2}, 1, \frac{7}{2}; z_j\right)$$

$$\Psi_j^{(2)} = \frac{k^2}{c_t^2 \alpha_j} F\left(1, \frac{1}{2}, \frac{7}{2}; z_j\right)$$

$$\Psi_{j\alpha}^{(3)} = -\frac{2k^4}{21\omega^2 \alpha_j} F\left(1, \frac{1}{2}, \frac{11}{2}; z_j\right)$$

$$\Psi_{j\alpha}^{(4)} = -\frac{2k_\alpha^4}{\omega^2 \alpha_j} F\left(1, \frac{1}{2}, \frac{7}{2}; z_j\right)$$

$$\Psi_{j\alpha}^{(5)} = \frac{2ik_\alpha k^4}{21\omega^2 \alpha_j^2} F\left(\frac{3}{2}, 1, \frac{11}{2}; z_j\right)$$

$$\Psi_{j\alpha}^{(6)} = \frac{4k_\alpha^2 k^2}{7\omega^2 \alpha_j} F\left(1, \frac{1}{2}, \frac{9}{2}; z_j\right)$$

$$\Psi_{j\alpha}^{(7)} = -\frac{4ik_\alpha^3 k^2}{7\omega^2 \alpha_j^2} F\left(\frac{3}{2}, 1, \frac{9}{2}; z_j\right)$$

$$\Psi_{j\alpha}^{(8)} = -\frac{5k_\alpha}{c_\alpha^2 \alpha_j} F\left(1, \frac{1}{2}, \frac{5}{2}; z_j\right)$$

$$\Psi_j^{(9)} = \frac{15ik_l^4}{4k\omega^2} \ln \frac{\alpha_j + ik}{\alpha_j - ik}$$

$$\Psi_{j\alpha}^{(10)} = \frac{4k_\alpha^2 k^4}{63\omega^2 \alpha_j^3} F\left(2, \frac{3}{2}, \frac{11}{2}; z_j\right)$$

$$\alpha_{1,2} = \frac{1 \mp i\delta_\beta}{a}, \quad \delta_\beta = 1 - ak_\beta \quad (\beta = l, t)$$

$$\{f_j\}_2^1 = f_1 - f_2, \quad \{f_\alpha\}\big|_t^l = f_l - f_t$$

$$A = -\frac{\pi R(0)\, a}{2i\rho_0}, \quad a\,|\theta_\beta| < 1, \quad \theta_\beta = -\operatorname{Im} k_\beta$$

Substituting L^* into the equations (1.16) and solving them for P, we obtain

$$P_{kn} = P_{kn}^{(l)} + P_{kn}^{(t)}, \; P_{kn}^{(l)} = P^{(l)}_{q_k q_n}, \; P_{kn}^{(t)} = P^{(t)}(\delta_{kn} - q_k q_n) \tag{2.8}$$
$$P^{(l)} = -[(L_1^* + 2L_2^*)k^2 - \rho_0\omega^2]^{-1}, \quad P^{(t)} = -[L_2^* k^2 - \rho_0 \omega^2]$$

From (2.7) it follows that $D_i^*(\omega, k)$, $L_i^*(\omega, k)$ are functions of k^2

$$D^*_{(\alpha)} = A\sum_{n=0}^{n} d_n^{(\alpha)} k^{2n} \quad (\alpha = 0, 1, 2) \tag{2.9}$$

$$D^*_{(0)} = D^*, \quad D^*_{(2)} = D_2^*, \quad D^*_{(1)} = D_1^*,$$

$$L^*_{(\alpha)} = \lambda_0^{(\alpha)} + A a_0^{(\alpha)-1} \sum_{n=0}^{\infty} c_n^{(\alpha)} k^{2n}$$
$$L_0^* = L^*, \quad L^*_{(1)} = L_1^* + 2L_2^*$$
$$a_0^{(\alpha)} = 1 - AT^{(\alpha)} d_0^{(\alpha)}, \quad a_n^{(\alpha)} = - AT^{(\alpha)} d_n^{(\alpha)} \quad (n = 1, 2, \ldots)$$
$$c_n^{(\alpha)} = d_n^{(\alpha)} - a_0^{(\alpha)-1} \sum_{k=0}^{n} a_k^{(\alpha)} c_{n-k}^{(\alpha)}, \quad a_0^{(l)} = 3a_0^{(0)} a_0^{(t)}$$

Substituting (2.9) into (2.8), we can write

$$P^{(\beta)}(k, \omega) = - [\varkappa_\beta (k^2) - \rho_0 \omega^2]^{-1} = - 2 \sum_{n=0}^{\infty} \frac{k_n^{(\beta)}}{\varkappa_\beta' (k_n^{(\beta)})(k^2 - k_n^{(\beta)2})} \tag{2.10}$$
$$\varkappa_\beta (k^2) = \sum_{n=1}^{\infty} s_n^{(\beta)} k^{2n}, \quad s_1^{(\beta)} = \lambda_0^{(\beta)} + A s_0^{(\beta)} a_0^{(\beta)-1}$$
$$s_n^{(\beta)} = A s_{n-1}^{(\beta)} a_0^{(\beta)-1}, \quad \lambda_0^{(t)} = \mu_0, \quad \lambda_0^{(l)} = \lambda_0 + 2\mu_0$$

Performing the inverse Fourier transformation in (2.8) with (2.10) taken into account, we obtain

$$\langle G_{ij} \rangle = - \frac{\delta_{ij}}{4\pi\rho} \sum_{n=0}^{\infty} \frac{k_n^{(t)}}{\varkappa_t' (k_n^{(t)})} \exp (ik_n^{(t)} \rho) + \tag{2.11}$$
$$\frac{\partial^2}{\partial x_i \partial x_j} \left\{ \frac{1}{4\pi\rho} \left(\sum_{n=0}^{\infty} \frac{k_n^{(\alpha)}}{\varkappa_\alpha' (k_n^{(\alpha)})} \exp (ik_n^{(\alpha)} \rho)_t{}^l \right) \right\}$$

3. Let us now turn our attention to computing the approximate values of $\langle G_{ij} \rangle$ at $n = 0, 1$. When $n = 0$ we obtain, with (2.10) taken into account,

$$k_0^{(\alpha)} = k_\alpha (1 + A c_0^{(\alpha)} a_0^{(\alpha)-1}), \quad k_\alpha = \frac{\omega}{c_\alpha}, \quad c_\alpha = \frac{\lambda_0^{(\alpha)}}{\rho_0} \tag{3.1}$$

The relations (3.1) yield the following asymptotic formulas depending on the wavelength of the waves in question:

$$ak_\alpha \ll 1 \ (\omega\tau_\alpha \ll 1, \ \tau_\alpha = a c_\alpha^{-1})$$

for the long waves with the case $k_0^{(\alpha)} = k_\alpha$ yielding a homogeneous medium with elastic coefficients λ_0, and μ_0;

$$ak_\alpha \approx 1 \ (\omega\tau_\alpha \approx 1), \quad | \delta_\alpha | \ll 1, \quad \delta_\alpha = 1 - \omega\tau_\alpha \quad (\alpha = l, t)$$
$$k_0^{(\alpha)} = k_\alpha \left(1 + \frac{\eta_0^{(\alpha)}}{\lambda_0^{(\alpha)}}\right)^{-1/2}, \quad \eta_0^{(t)} = 2N \left(\frac{\delta_l}{c_l^2} + \frac{\delta_t}{c_t^2}\right)$$
$$\eta_0^{(l)} = N \left(\frac{27\delta_l}{c_l} + \frac{28\delta_t}{c_t}\right), \quad N = \frac{\pi R(0)}{15\rho_0}$$

for the waves with wavelengths of the order of the inhomogeneities, and

$$ak_\alpha \gg 1 \quad (\omega\tau_\alpha \gg 1)$$
$$k_0^{(\alpha)} = ik_\alpha (\lambda_0^{(\alpha)} T^{(\alpha)})^{1/2} (1 - \lambda_0^{(\alpha)} T^{(\alpha)})^{-1}$$

for the short waves.

Taking into account (2.11) and (2.12), we obtain for $n = 0$

$$\langle G_{(\rho)}{}^{(\alpha)}\rangle = -T^{(\alpha)}\,[4\pi\rho\,(\lambda_0{}^{(\alpha)}T^{(\alpha)} - 1)]^{-1} \exp\,(ik_0{}^{(\alpha)}\rho] \tag{3.2}$$

where $\langle G_{(\rho)}{}^{(\alpha)}\rangle$ is the original function of $P^{(\alpha)}\,(\omega, k)$. When $n = 1$, we have with (2.10) taken into account,

$$k_0^{(\alpha)2} = 2\rho_0\omega^2\,[S_0^{(\alpha)} + S_1^{(\alpha)}]^{-1}, \quad k_1^{(\alpha)2} = -\frac{Ac_1^{(\alpha)}}{2a_0^{(\alpha)}}\,[S_0^{(\alpha)} + S_1^{(\alpha)}] \tag{3.3}$$

$$S_0^{(\alpha)} = \lambda_0^{(\alpha)} + Ac_0^{(\alpha)}a_0^{(\alpha)-1}, \quad S_1^{(\alpha)} = (S_0^{(\alpha)2} + 4\rho_0\omega^2 Ac_1^{(\alpha)}a^{(\alpha)-1})^{1/2}$$

Investigation of the asymptotics of the formulas (3.3) with respect to the wavelengths (frequencies) can be reduced to the problem of finding the corresponding expressions for $c_0{}^{(\alpha)}$, $c_1{}^{(\alpha)}$ $k_0{}^{(\alpha)}$, $k_1{}^{(\alpha)}$. We have

$$\begin{aligned}
&k_0^{(\alpha)} = k_\alpha\left(1 + i\,\frac{\rho_0 a\omega^3}{2\lambda_0^{(\alpha)2}}\,\theta_{(1)2}^{(\alpha)}\right)\\
&k_1^{(\alpha)} = {}^1/_2\left(\frac{\lambda_0^{(\alpha)}}{\theta_{(1)1}^{(\alpha)3}}\right)^{1/2}(a\omega\theta_{(1)2}^{(\alpha)} + 2i\theta_{(1)1}^{(\alpha)})\\
&\theta_{(1)1}^{(l)} = -\frac{2Na^2}{21}\left(\frac{31}{c_l^2} + \frac{3}{c_t^2}\right), \quad \theta_{(1)2}^{(l)} = -\frac{2Na^2}{7}\left(\frac{13}{c_l^3} + \frac{1}{c_t^3}\right)\\
&\theta_{(1)1}^{(t)} = -\frac{Na^2}{7}\left(\frac{4}{c_l^2} + \frac{3}{c_t^2}\right), \quad \theta_{(1)2}^{(t)} = \frac{2Na^2}{7}\left(\frac{4}{c_l^3} + \frac{3}{c_t^3}\right)\\
&\langle G_{(\rho)}^{(\alpha)}\rangle = -\frac{1}{4\pi\rho}\,[(\lambda_0^{(\alpha)} + 2ia\omega^3 c_\alpha^{-2}\theta_{(1)2}^{(\alpha)})^{-1}\exp\,(ik_0^{(\alpha)}\rho) -\\
&\qquad (\lambda_0^{(\alpha)})^{-1}\exp\,(ik_1^{(\alpha)}\rho)]
\end{aligned} \tag{3.4}$$

for long waves,

$$\begin{aligned}
&k_0^{(\alpha)} = k_\alpha\,[1 - (2\lambda_0^{(\alpha)})^{-1}(\eta_0^{(\alpha)} + \eta_{(1)2}^{(\alpha)}a^{-2})]\\
&k_1^{(\alpha)} = \left(\frac{\lambda_0^{(\alpha)}}{2}\right)^{1/2}(-\eta_{(1)1}^{(\alpha)} + i\eta_{(1)}^{(\alpha)2})(\eta^{(\alpha)}\eta_1^{(\alpha)})^{-1}\\
&\eta^{(\alpha)} = (\eta_{(1)1}^{(\alpha)2} + \eta_{(1)2}^{(\alpha)2}), \quad \eta_1^{(\alpha)} = (\eta^{(\alpha)} - \eta_{(1)2}^{(\alpha)})^{1/2}, \eta_0^{(\alpha)} = Ac_0^{(\alpha)}\\
&\eta_{(1)1}^{(t)} = -\frac{4Na^2}{7}\left(\frac{5\delta_l}{c_l^2} + \frac{9\delta_t}{c_t^2}\right), \quad \eta_{(1)2}^{(t)} = \frac{4Na^2}{7}\left(\frac{4\delta_l}{c_l^2} + \frac{3\delta_t}{c_t^2}\right)\\
&\eta_{(1)1}^{(l)} = -\frac{Na^2}{21}\left(\frac{831\delta_l}{c_l^2} + \frac{156\delta_t}{c_t^2}\right), \quad \eta_{(1)2}^{(l)} = \frac{8Na^2}{7}\left(\frac{13\delta_l}{c_l^2} + \frac{\delta_t}{c_t^2}\right)\\
&\langle G_{(\rho)}^{(\alpha)}\rangle = -\frac{1}{4\pi\rho}\,[(\lambda_0^{(\alpha)} + 2ik_\alpha\eta_{(1)2}^{(\alpha)})^{-1}\exp\,(ik_0^{(\alpha)}\rho) -\\
&\qquad (\lambda_0^{(\alpha)})^{-1}\exp\,(ik_1^{(\alpha)}\rho)]
\end{aligned} \tag{3.5}$$

for the waves with wavelengths of the order of the inhomogeneities, and

$$\begin{aligned}
&k_0{}^{(\alpha)} = ik_\alpha\,(\lambda^{(\alpha)}T^{(\alpha)})^{1/2}\,(1 - \lambda_0{}^{(\alpha)}T^{(\alpha)})^{-1}\\
&c_1{}^{(\alpha)} = c_2{}^{(\alpha)} = \ldots = 0, \quad (T^{(l)})^{-1} = T^{-1} + {}^4/_3\,(T^{(t)})^{-1}\\
&\langle G_{(\rho)}{}^{(\alpha)}\rangle = T^{(\alpha)}\,[4\pi\rho\,(1 - \lambda_0{}^{(\alpha)}T^{(\alpha)})]^{-1}\exp\,(ik_0{}^{(\alpha)}\rho)
\end{aligned} \tag{3.6}$$

for short waves.

For the formulas (2.12) with (3.1) — (3.6) taken into account, it follows that an inhomogeneity leads to the appearance of exponentially decaying terms in the expression for the averaged function. The expression for short waves contains only the decaying terms. When exact operator relations are used, the dispersion equations become incorrect [10] in the short wave region $ak_\alpha \gg 1$, and ray methods have to be used in investigating the fields [3]. We note that in the cases of long and short waves the series (2.10) are truncated, while for $ak_\alpha \approx 1$ all terms in the series are proportional to δ_α, and for this reason the expression (3.5) contains only the first two terms. In the formula for $k_0^{(\alpha)}$ in (3.4) we neglect, in the real part, the term proportional to $(ak_\alpha)^2$. In selecting the roots of (3.3) we use the condition that $k_0^{(\alpha)}$ tends to k_α on passing to the homogeneous medium $(R_{(0)} \to 0)$. Then the first term in the expression for $\langle G_{(\rho)}^{(\alpha)} \rangle$ becomes the Green's function for a homogeneous medium, and the second term vanishes. In the case of the remaining two roots $k_1^{(\alpha} \to k_\alpha$ as $R(0) \to 0$. The first term in $\langle G_{(\rho)}^{(\alpha)} \rangle$ vanishes, and the second term becomes the Green's function for the homogeneous medium.

The correlation function (2.3) represents a product of the following functions: $R_1(\rho)=R_1(0) \exp(-\tau)$ characterizes completely disorganized structures [11] and $R_2(\rho) = R_2(0)\tau^{-1} \sin \tau$ determines the Markovian field of the coefficients $\gamma(x)$ [12, 13].

The presence of negative correlation means that the function (2.3) describes the fields $\gamma(x)$ which vary arbitrarily rapidly over the length of order $2\pi a$ (i.e. they assume values of different sign sufficiently often). A correlation dependence of the type (2.3) has been detected in experiments [14]. The method of computing the Green's tensor discussed here can be used with any, exponentially-trigonometric correlation function which can be represented in the exponential form.

In particular, for $R(\rho) = R(0) \exp(-\tau)$ the expressions (2.7) simplify in the following manner: 1) instead of $\alpha_j (j = 1,2)$ we write $\alpha_2 = a^{-1}(1 - iak_\beta)$, 2) the difference in j disappears, and $\{f_j\}_2^1 = f_2$, 3) $A = -\pi R(0)(2\rho_0)^{-1}$. Corresponding changes take place in (3.1) — (3.4), and the qualitative character of the dependence of $k^{(\alpha)}$ on ω remains unaltered. Formulas (3.5) will disappear, since in this case the small parameter $\delta_\alpha = 1 - ak_\alpha$ will not be present in the relations (2.10).

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